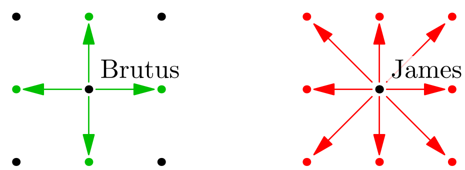
2026 Utah Math Olympiad April 11, 2026

1. Find (with proof) all ordered pairs of integers (n, k), with $n \geq 0$ and $1 \leq k \leq 10$, such that

$$n! + 1 = (kn + 1)^2.$$

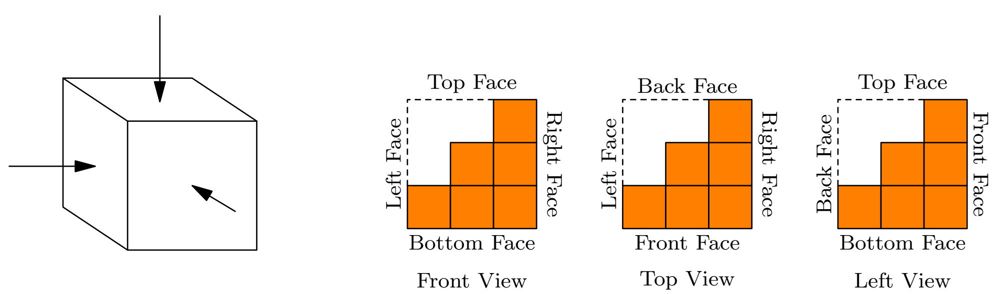
2. James is trying to catch his dog, Brutus, in his yard, which is an $N \times N$ grid of lattice points. James starts in the top-left corner, and Brutus starts at a lattice point different from James, chosen at random. In each round, Brutus moves first, moving to a neighboring lattice point in a horizontal or vertical direction. Then James moves, moving to a neighboring lattice point in a horizontal, vertical, or diagonal direction, as shown below. Both James and Brutus must remain within the yard.



Possible moves from a lattice point

If at any point, James and Brutus are on the same lattice point, James catches Brutus. What is the expected value of the number of moves it will take James to catch Brutus if Brutus plays optimally to remain free as long as possible, and James plays optimally so as to catch Brutus as soon as possible?

3. Orange unit cubes are placed in a $3 \times 3 \times 3$ cubical grid such that the views looking from the front, from the top, and from the left are each the following picture, as shown below.



The unit cubes are not constrained by gravity and may be floating in mid-air. How many arrangements of unit cubes are possible?

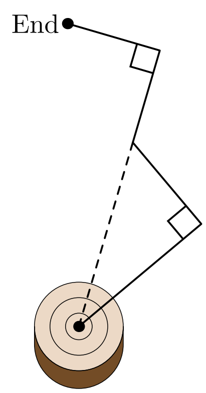
4. An increasing sequence of nonnegative integers $a_0, a_1, a_2, a_3, \ldots$ has the *sum-of-two property* if every nonnegative integer n can be written in the form $a_i + a_j$ for some (not necessarily distinct) i and j. For example, the sequence $0, 1, 3, 5, 7, \ldots$ (continuing with all odd numbers) has the sum-of-two property.

   Prove or disprove that if $a_0, a_1, a_2, \ldots$ has the sum-of-two property, then there exists a constant K such that $a_i \leq Ki$ for all i.

5. Fiona is standing on a tree stump in a large swamp holding $2n$ straight wooden logs of distinct lengths. Initially, she faces in any direction, and then she performs the following sequence of actions n times:

    1. She turns to face directly away from the center point of the stump.
    2. She places down a log at her feet going directly in front of her, and walks to the end of the log.
    3. She turns $90°$ to the left.
    4. She places down a new log at her feet going directly in front of her, and walks to the end of the log.

    In what order should Fiona place the logs to get as far away as possible from the stump after placing all the logs? Give all orderings of the logs that maximize this distance. For example, if the lengths of the logs are $x_1 < x_2 < \cdots < x_{2n}$ and she places the logs in increasing order of length, then the ordering is $(x_1, x_2, \ldots, x_{2n})$. A sample path is shown when $n = 2$.



6. Each of the (distinct) circles $\omega_1, \omega_2, \omega_3, \omega_4$ is tangent to the other three. Let O_i be the center of ω_i. There exists a unique point P_1, distinct from O_1, in the exterior of ω_2, ω_3, ω_4 that has the same power with respect to these three circles. Points P_2, P_3, P_4 are characterized similarly, so that P_i ($\neq O_i$) is the unique point having a constant power relative to ω_j when $j \neq i$ (and in the exterior of the ω_j's). Show that the sixteen lines $\overleftrightarrow{O_iP_j}$ can be partitioned into four subsets S_1, S_2, S_3, S_4 such that for each k,

    (a) Either all lines in S_k are concurrent, or all lines in S_k are parallel.

    (b) The eight points O_1, O_2, O_3, O_4, P_1, P_2, P_3, P_4 lie in the union of the lines of S_k.

    (Note: If point Z is outside circle ω, and A, B are points on ω such that Z, A, B are collinear, then the product of distances $ZA \cdot ZB$ is called the **power** of point Z with respect to circle ω. This number depends only on Z and the circle ω—if Z, C, D are collinear and C, D lie on ω, then $ZC \cdot ZD = ZA \cdot ZB$.)